Minimal and Reduced Reversible Automata

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Joint work with

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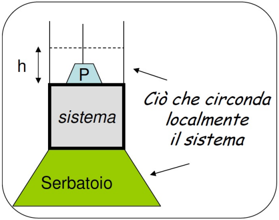
17 Giugno 2016

Reversible automata

Motivations and related work

- Reversibility is a fundamental principle in physics
- Reversible transformations: (ideal) reversible process in thermodynamics
- Reversible processes imply no losses of information
- Logical irreversibility is associated with physical irreversibility and implies a certain amount of heat generation [Landauer '61]



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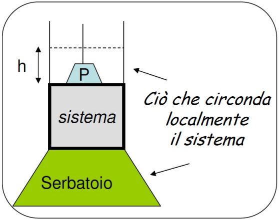

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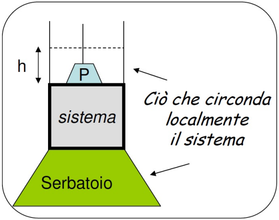

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Reversible automata

Motivations and related work

Reversibility in different computational devices:

- Each Turing machine can be simulated by a reversible machine [Bennet '73]
- Each deterministic machine can be simulated by a reversible machine which uses the same amount of space [Lange&McKenzie&Tapp '00]
- In the case of a constant amount of space, this implies that each regular language is accepted by a *reversible two-way deterministic finite automaton* [Kondacs '97]

In the case of *one-way* automata the situation is different

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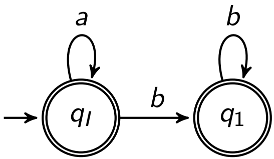
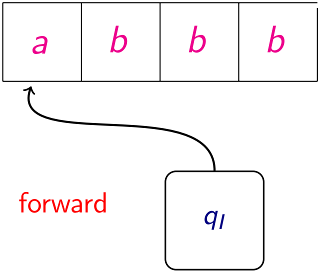
The regular language a^*b^* cannot be accepted
by any reversible automaton [Pin '92]





The class of languages accepted by *reversible automata*
is a proper subclass of the class of regular languages

The regular language $a^* b^*$ cannot be accepted
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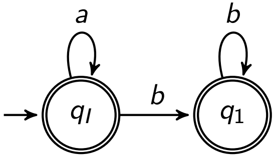

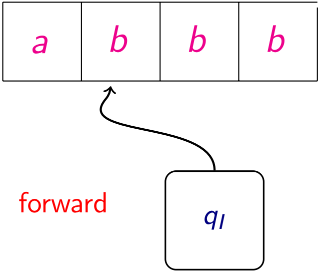

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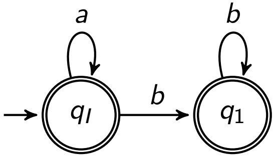

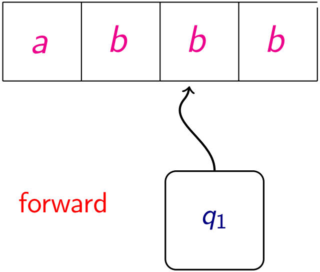

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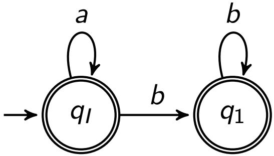

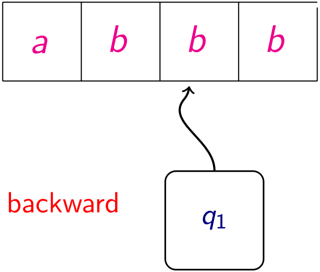

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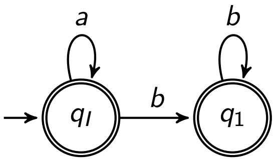

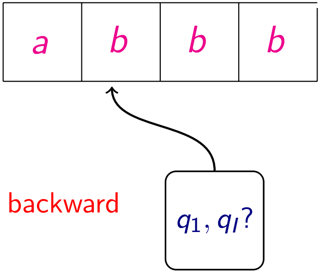

The class of languages accepted by *reversible automata* is a proper subclass of the class of regular languages

Reversible automata

Motivations and related work

There are some different notions of reversible automata:

- *One initial* and only *one final state* [Angluin '82]
- A *set of accepting states* and a *set of initial states* [Pin '92]
- *One initial* state and a *set of final states* [Holzer&Jakobi&Kutrib '15]

Reversible automata: preliminaries

Let $A = (Q, \Sigma, \delta, q_I, F)$ be a *deterministic automaton* (DFA)

We assume:

- Partial *transition function* δ
- Only useful states (no dead state)
- Unique initial state q_I and possibly many accepting states

Reverse transiton function

The *reverse* transition function $\delta^R : Q \times \Sigma \to 2^Q$ associates with each state $r \in Q$ and letter $a \in \Sigma$ the set of states from which r can be reached by reading a, i.e.,

$$\delta^R(r, a) = \{q \in Q \mid \delta(q, a) = r\}$$

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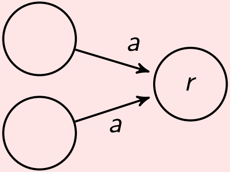
Reversible automata: preliminaries

Irreversible state

A state $r \in Q$ is said to be *irreversible* when there are at least two transitions on the same letter $a \in \Sigma$ entering r, i.e.,

$$\#\delta^R(r, a) \geq 2$$



otherwise r is *reversible*

Reversible automata: preliminaries

- A DFA is said to be *reversible* (REV-DFA) when each state is reversible
- A *language* is *reversible* when there exists a REV-DFA accepting it

reversible language

$a^* b a^*$

not reversible language

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How to decide the reversibility of a given regular language?

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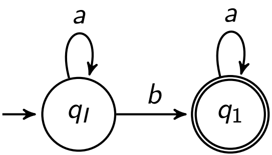

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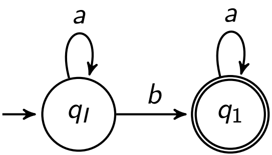

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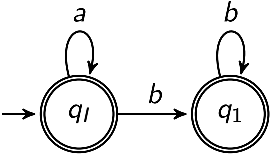

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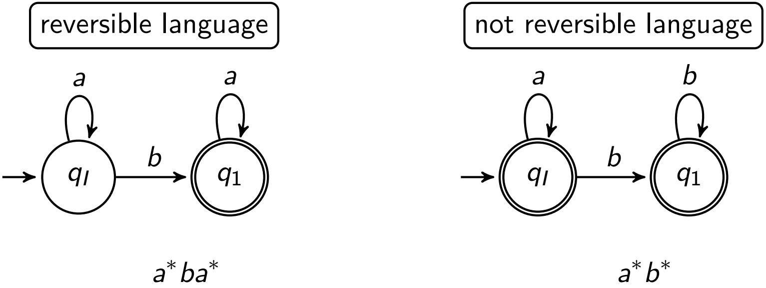


How to decide the reversibility of a given regular language?

Reversible automata: preliminaries

Some key notions

Let $\mathcal{C}$ be the family of reversible automata accepting a given language L and $A \in \mathcal{C}$. The automaton A is:

- *reduced* in $\mathcal{C}$ if every automaton obtained from A by merging some equivalent states does not belong to $\mathcal{C}$
- *minimal* in $\mathcal{C}$ if each automaton in $\mathcal{C}$ has at least as many states as A
- the *minimum* in $\mathcal{C}$ if it is the unique (up to isomorphism) minimal automaton in $\mathcal{C}$

Reversible automata: preliminaries

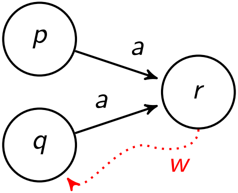
A structural characterization of reversible languages

Let L be a regular language and
$M = (Q, \Sigma, \delta, q_I, F)$ be the minimum DFA accepting L

Theorem ([Holzer&Jakobi&Kutrib '15])

The language L is accepted by a REV-DFA if and only if there do not exist useful states $p, q \in Q$, a letter $a \in \Sigma$, and a string $w \in \Sigma^*$ such that $p \neq q$, $\delta(p, a) = \delta(q, a)$, and $\delta(q, aw) = q$.

Forbidden pattern:



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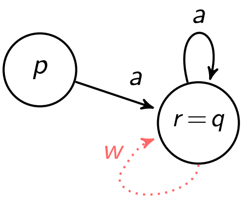

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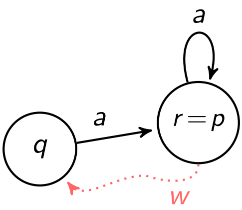
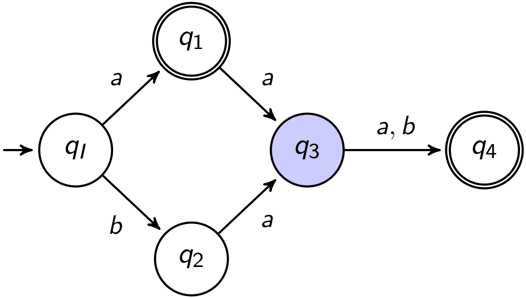


Reversible automata: preliminaries

An algorithm: from the minimum DFA to an equivalent minimal REV-DFA

$$L = \{a, aaa, aab, baa, bab\}$$



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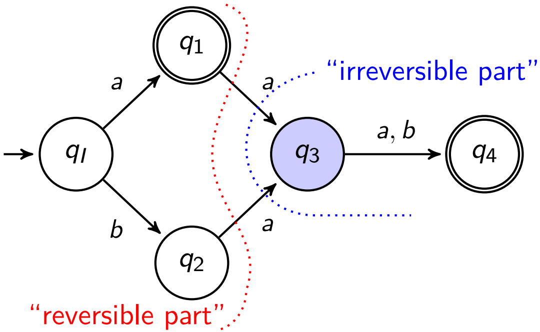

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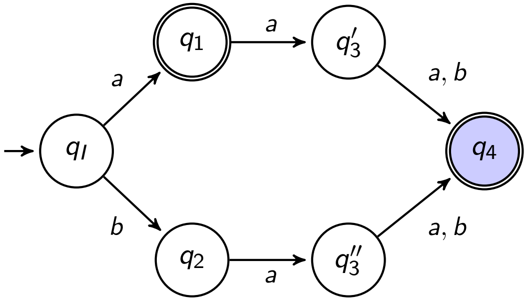

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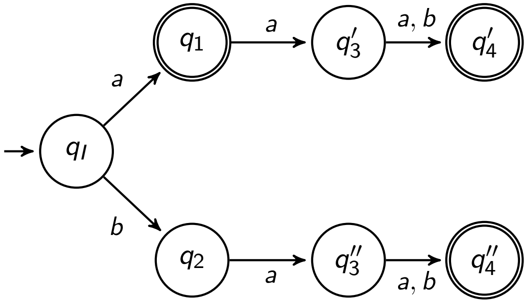

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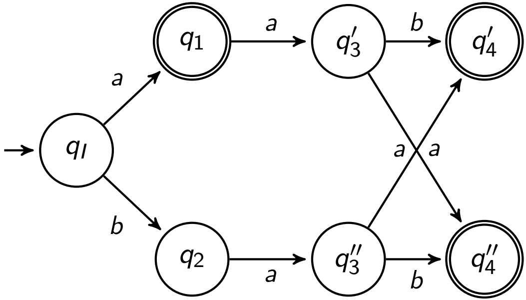


Another nonisomorphic minimal REV-DFA accepting L

Our Goal

Here we want to go beyond ...

We investigate the *existence of minimal and reduced* REV-DFAs.

Minimal Reversible Automata

Minimal Reversible Automata

A characterization of languages having several different minimal REV-DFAs

Let us fix a reversible language L and
the minimum DFA $M = (Q, \Sigma, \delta, q_I, F)$ accepting it

Theorem

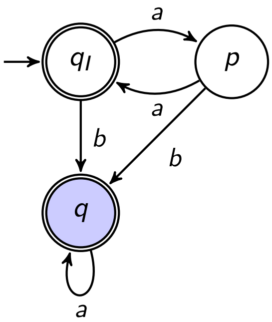
The following statements are equivalent:

1. *There exists a state $q \in Q$ in the irreversible part such that $\delta^R(q, a) \neq \emptyset$, $\delta^R(q, b) \neq \emptyset$, for two symbols $a, b \in \Sigma$, with $a \neq b$.*
2. *There exist at least two minimal nonisomorphic REV-DFAs accepting L.*

Minimal Reversible Automata

An example: $L = (aa)^* + a^*ba^*$

A minimum DFA M accepting a reversible language L



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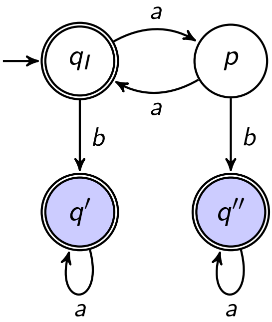
An equivalent minimal REV-DFA A', obtained applying the previous algorithm



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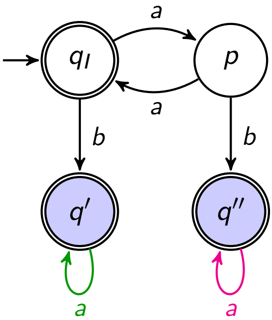
The idea is to use the set of states as in A' and to modify only the transitions which simulates the transitions that in M enter the state q



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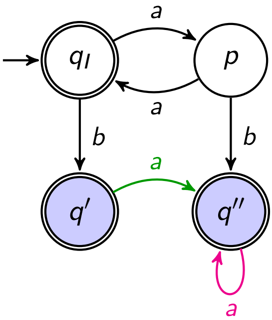

Minimal Reversible Automata

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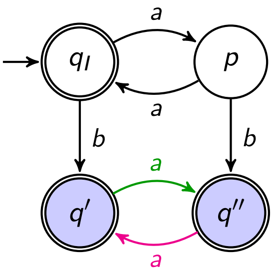
An equivalent minimal REV-DFA A'' nonisomorphic to A'



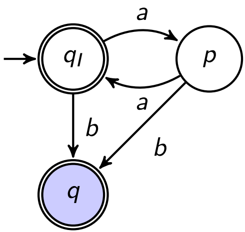
Minimal Reversible Automata

A characterization of languages having a unique minimum REV-DFA

Corollary

Let $M = (Q, \Sigma, \delta, q_I, F)$ be the minimum DFA accepting a reversible language L.
There exists a unique (up to isomorphism) minimal REV-DFA accepting L if and only if for each $q \in Q$ in the irreversible part, all the transitions entering in q are on the same symbol.

The minimum DFA accepting $L = (aa)^* + a^*b$:



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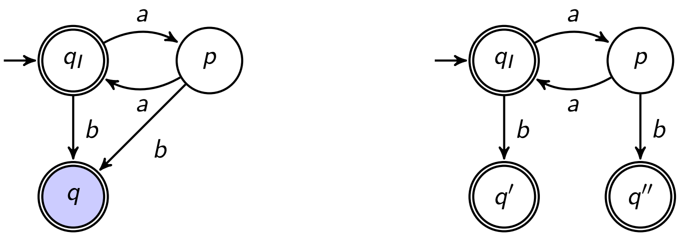
The minimum DFA and the equivalent minimum REV-DFA:



Minimal Reversible Automata

Observations

Minimal $\equiv$ Minimum number of states

$\Rightarrow$ Trying to merge two equivalent states in a minimal REV-DFA A, the resulting automaton is not reversible, i.e., A is *reduced*

However,

there exist REV-DFAs which are reduced but not minimal!

Reduced Reversible Automata

Reduced Reversible Automata

A condition for the existence of infinitely many reduced REV-DFAs

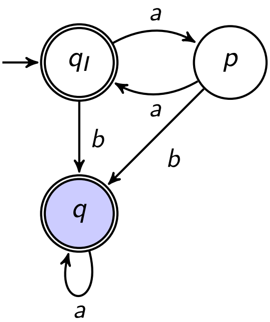
Let $M = (Q, \Sigma, \delta, q_I, F)$ be the minimum DFA accepting a reversible language L

Theorem

If M contains a state q in the irreversible part and the language accepted by computations starting in q is infinite, then there exist infinitely many nonisomorphic reduced REV-DFAs accepting L.

Reduced Reversible Automata

An example: $L = (aa)^* + a^*ba^*$



The minimum DFA

Reduced Reversible Automata

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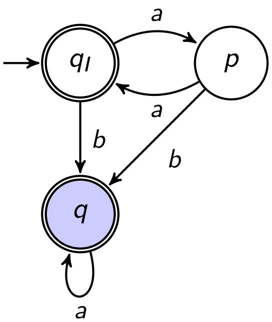

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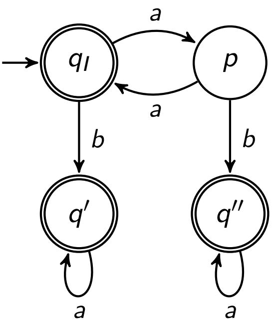

An equivalent minimal REV-DFA

Reduced Reversible Automata

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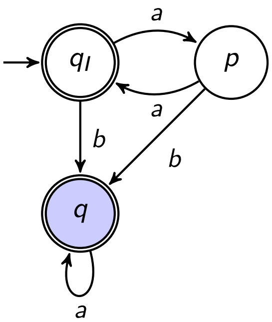

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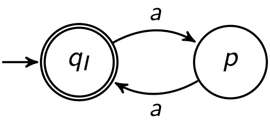

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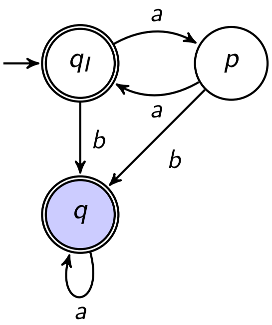

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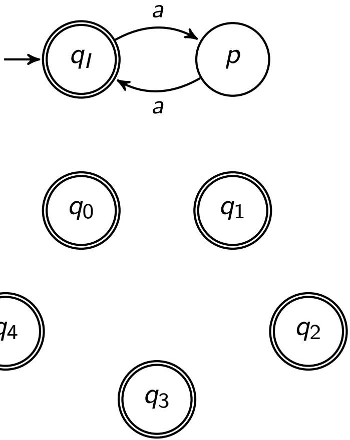

A prime number N of copies
(here $N = 5$)

Reduced Reversible Automata

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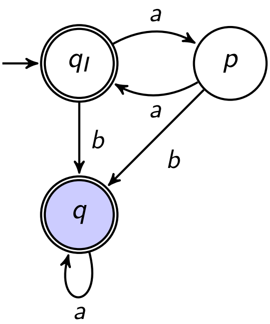
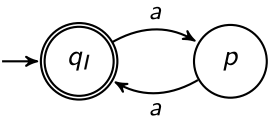
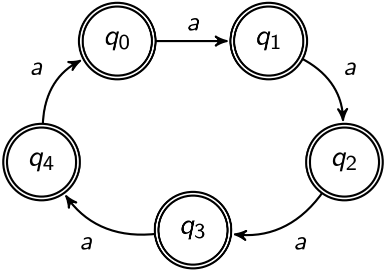


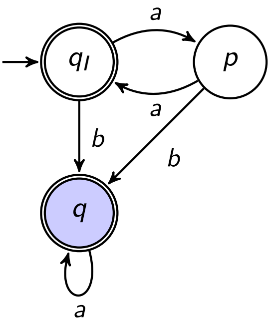
The minimum DFA



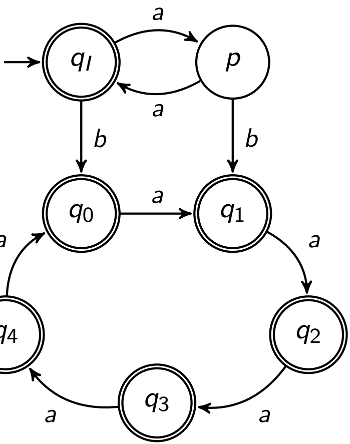


Reduced Reversible Automata

An example: $L = (aa)^* + a^*ba^*$



The minimum DFA

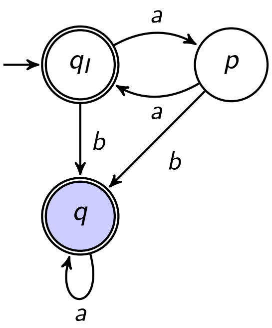


An equivalent reduced REV-DFA

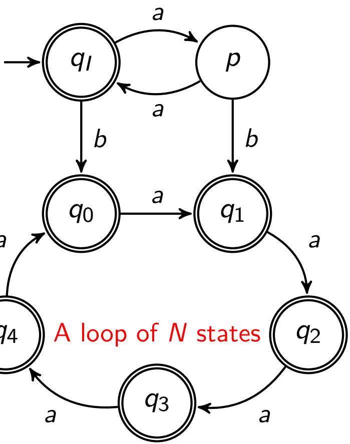
Reduced Reversible Automata

An example: $L = (aa)^* + a^* b a^*$

If and only if N is prime, we get a reduced automaton



The minimum DFA



An equivalent reduced REV-DFA

Conclusion

Our contributions

- A characterization of the languages having several different minimal reversible automata, in terms of the structure of the minimum DFA
- A characterization of the languages having a unique minimum REV-DFA
- A sufficient condition for the existence of infinitely many reduced REV-DFAs accepting a same reversible language

Open problem

- The condition for the existence of infinitely many reduced REV-DFAs accepting a same reversible language is sufficient

but it is not necessary

If the minimum DFA does not contain any loop in the irreversible part, it could be possible to build infinitely many reduced REV-DFAs

Problem: left open!

To find a characterization of infinitely many reduced REV-DFAs

Open problem

- The condition for the existence of infinitely many reduced REV-DFAs accepting a same reversible language is sufficient

but it is not necessary

If the minimum DFA does not contain any loop in the irreversible part, it could be possible to build infinitely many reduced REV-DFAs

Problem: left open!

To find a characterization of infinitely many reduced REV-DFAs

Future works

Some possible future investigations:

- It could be interesting to study other models:
  - Unique initial state and a unique final state
  - A set of initial states and a set of accepting states
- To study the closure properties of reversible languages under some regular operations

Thank you for your attention.