Minimal and Reduced Reversible Automata

Giovanna J. Lavado, Giovanni Pighizzini, Luca Prigioniero

Dipartimento di Informatica
Università degli Studi di Milano, Italy

ICTCS 2016
Università del Salento, Lecce, Italy

September 7-9, 2016

Reversible automata

Motivations and related work

- Reversibility is a fundamental principle in physics
- Reversible transformations: (ideal) reversible process in thermodynamics
- Reversible processes imply no loss of information
- Logical irreversibility is associated with physical irreversibility and implies a certain amount of heat generation [Landauer '61]

In order to avoid power dissipation and reduce the overall power consumption, the possibility of realizing reversible machines looks appealing!

Reversible automata

Motivations and related work

- Reversibility is a fundamental principle in physics
- Reversible transformations: (ideal) reversible process in thermodynamics
- Reversible processes imply no loss of information
- Logical irreversibility is associated with physical irreversibility and implies a certain amount of heat generation [Landauer '61]

In order to avoid power dissipation and reduce the overall power consumption, the possibility of realizing reversible machines looks appealing!

Reversible automata

Motivations and related work

- Reversibility is a fundamental principle in physics
- Reversible transformations: (ideal) reversible process in thermodynamics
- Reversible processes imply no loss of information
- Logical irreversibility is associated with physical irreversibility and implies a certain amount of heat generation [Landauer '61]

In order to avoid power dissipation and reduce the overall power consumption, the possibility of realizing reversible machines looks appealing!

Reversible automata

Motivations and related work

Reversibility in different computational devices:

- Each Turing machine can be simulated by a reversible machine [Bennet '73]
- Each deterministic machine can be simulated by a reversible machine which uses the same amount of space [Lange&McKenzie&Tapp '00]
- This implies that each regular language is accepted by a *reversible two-way deterministic finite automaton* [Kondacs '97]

In the case of *one-way* automata the situation is different

Reversible automata

Motivations and related work

Reversibility in different computational devices:

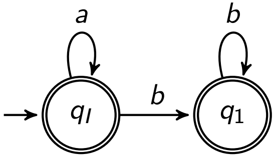
- Each Turing machine can be simulated by a reversible machine [Bennet '73]
- Each deterministic machine can be simulated by a reversible machine which uses the same amount of space [Lange&McKenzie&Tapp '00]
- This implies that each regular language is accepted by a *reversible two-way deterministic finite automaton* [Kondacs '97]

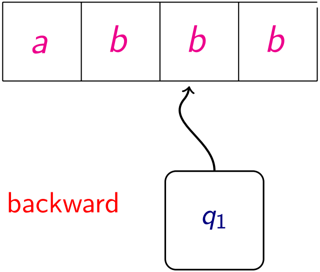
In the case of *one-way* automata the situation is different

Reversible automata

Motivations and related work

The regular language a^*b^* cannot be accepted by *any* reversible automaton [Pin '92]



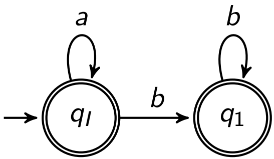


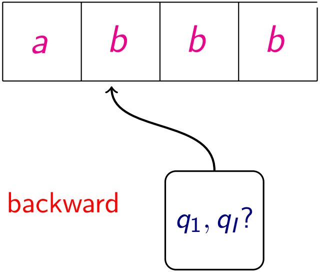
The class of languages accepted by *reversible automata* is a proper subclass of the class of regular languages

Reversible automata

Motivations and related work

The regular language a^*b^* cannot be accepted by *any* reversible automaton [Pin '92]





The class of languages accepted by *reversible automata* is a proper subclass of the class of regular languages

Reversible automata

Motivations and related work

There are some different notions of reversible automata:

- *One initial* and only *one final state* [Angluin '82]
- A *set of initial states* and a *set of final states* [Pin '92]
- *One initial* state and a *set of final states* as in classical automata

  [Holzer&Jakobi&Kutrib '15]

Reversible automata: preliminaries

Let $A = (Q, \Sigma, \delta, q_I, F)$ be a *deterministic automaton* (DFA)

We assume:

- Partial *transition function* δ
- Only useful states (no dead state)
- Unique initial state q_I and possibly many accepting states

Reverse transiton function

$$\delta^R : Q \times \Sigma \to 2^Q$$

associates with each state $r \in Q$ and letter $a \in \Sigma$ the set of states from which r can be reached by reading a, i.e.,

$$\delta^R(r, a) = \{q \in Q \mid \delta(q, a) = r\}$$

Reversible automata: preliminaries

Let $A = (Q, \Sigma, \delta, q_I, F)$ be a *deterministic automaton* (DFA)

We assume:

- Partial *transition function* δ
- Only useful states (no dead state)
- Unique initial state q_I and possibly many accepting states

Reverse transiton function

$$\delta^R : Q \times \Sigma \to 2^Q$$

associates with each state $r \in Q$ and letter $a \in \Sigma$ the set of states from which r can be reached by reading a, i.e.,

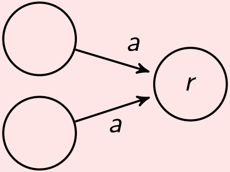
$$\delta^R(r, a) = \{q \in Q \mid \delta(q, a) = r\}$$

Reversible automata: preliminaries

Irreversible state

A state $r \in Q$ is said to be *irreversible* when there are at least two transitions on the same letter $a \in \Sigma$ entering r, i.e.,

$$\#\delta^R(r, a) \geq 2$$



otherwise r is *reversible*

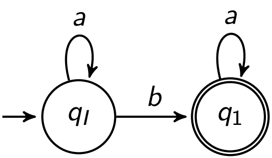
Reversible automata

- A DFA is said to be *reversible* (REV-DFA) when *each* state is reversible

Reversible automata

- A DFA is said to be *reversible* (REV-DFA) when *each* state is reversible

reversible automaton



a^*ba^*

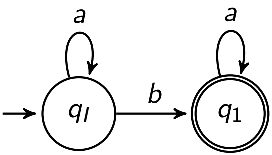
not reversible automaton

a^*b^*

Reversible automata

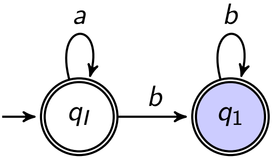
- A DFA is said to be *reversible* (REV-DFA) when *each* state is reversible

reversible automaton



$a^* b a^*$

not reversible automaton



$a^* b^*$

Reversible languages

- A *language* is *reversible* when there *exists* a REV-DFA accepting it

$a^* b^*$ is not a reversible language

How to decide the reversibility of a given regular language?

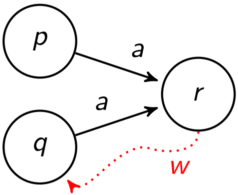
Reversible languages: a structural characterization

- L regular language
- $M = (Q, \Sigma, \delta, q_I, F)$ minimum DFA accepting L

Theorem ([Holzer&Jakobi&Kutrib '15])

The language L is reversible if and only if there do not exist useful states $p, q \in Q$, a letter $a \in \Sigma$, and a string $w \in \Sigma^*$ such that $p \neq q$, $\delta(p, a) = \delta(q, a)$, and $\delta(q, aw) = q$.

Forbidden pattern:



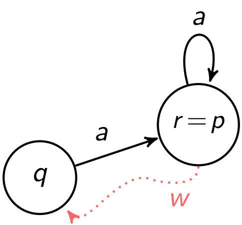
Reversible languages: a structural characterization

- L regular language
- $M = (Q, \Sigma, \delta, q_I, F)$ minimum DFA accepting L

Theorem ([Holzer&Jakobi&Kutrib '15])

The language L is reversible if and only if there do not exist useful states $p, q \in Q$, a letter $a \in \Sigma$, and a string $w \in \Sigma^*$ such that $p \neq q$, $\delta(p, a) = \delta(q, a)$, and $\delta(q, aw) = q$.

Forbidden pattern:



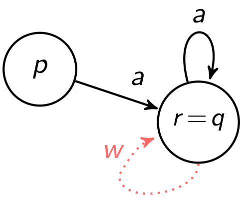
Reversible languages: a structural characterization

- L regular language
- $M = (Q, \Sigma, \delta, q_I, F)$ minimum DFA accepting L

Theorem ([Holzer&Jakobi&Kutrib '15])

The language L is reversible if and only if there do not exist useful states $p, q \in Q$, a letter $a \in \Sigma$, and a string $w \in \Sigma^*$ such that $p \neq q$, $\delta(p, a) = \delta(q, a)$, and $\delta(q, aw) = q$.

Forbidden pattern:



Reversible languages: a structural characterization

- L regular language
- $M = (Q, \Sigma, \delta, q_I, F)$ minimum DFA accepting L

Theorem ([Holzer&Jakobi&Kutrib '15])

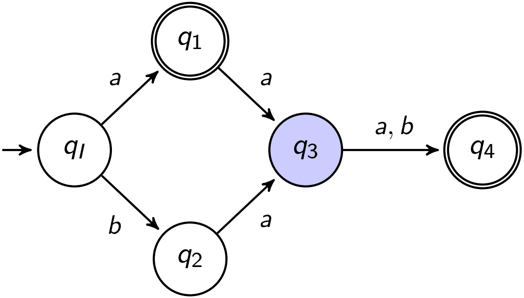
The language L is reversible if and only if there do not exist useful states $p, q \in Q$, a letter $a \in \Sigma$, and a string $w \in \Sigma^*$ such that $p \neq q$, $\delta(p, a) = \delta(q, a)$, and $\delta(q, aw) = q$.

There exists an algorithm that converts the minimum DFA accepting a reversible language into a REV-DFA

[Holzer&Jakobi&Kutrib '15]

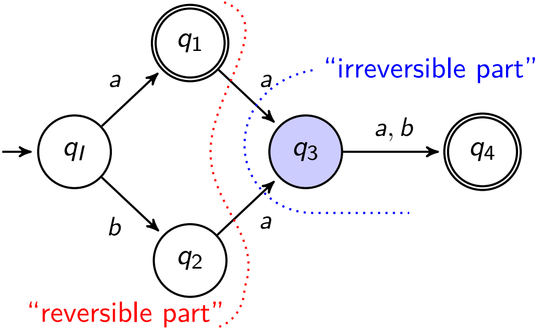
From the minimum DFA to an equivalent REV-DFA

The minimum DFA M accepting $L = \{a, aaa, aab, baa, bab\}$



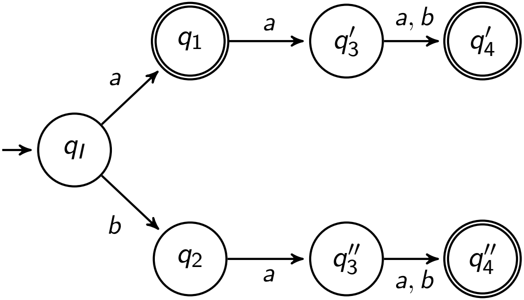
From the minimum DFA to an equivalent REV-DFA

The minimum DFA M accepting $L = \{a, aaa, aab, baa, bab\}$



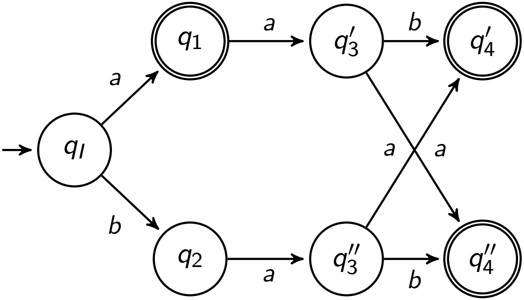
From the minimum DFA to an equivalent REV-DFA

A minimal REV-DFA accepting $L = \{a, aaa, aab, baa, bab\}$



From the minimum DFA to an equivalent REV-DFA

In general a minimal REV-DFA is not unique



Another nonisomorphic minimal REV-DFA accepting L

Our Goal

Here we want to go beyond ...

We investigate the *existence of many minimal and reduced* REV-DFA*s*.

Minimal Reversible Automata

Minimal Reversible Automata

A characterization of languages having several different minimal REV-DFAs

We obtained the following characterization:

Theorem

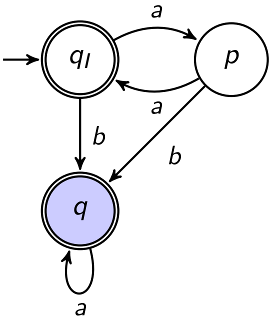
Let $M = (Q, \Sigma, \delta, q_I, F)$ be the minimum DFA *accepting a reversible language L. The following statements are equivalent:*

1. *There exist at least two minimal nonisomorphic* REV-DFAs *accepting L.*
2. *There exists a state $q \in Q$ in the irreversible part such that $\delta^R(q, a) \neq \emptyset$, $\delta^R(q, b) \neq \emptyset$, for some $a, b \in \Sigma$, with $a \neq b$.*

Minimal Reversible Automata

An example: $L = (aa)^* + a^*ba^*$

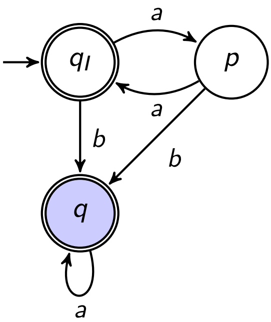
A minimum DFA M accepting a reversible language L



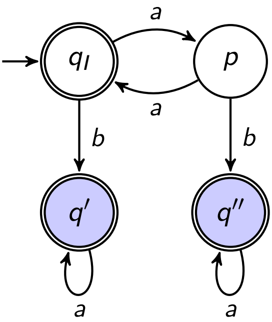
Minimal Reversible Automata

An example: $L = (aa)^* + a^*ba^*$

An equivalent minimal REV-DFA A', obtained by the algorithm



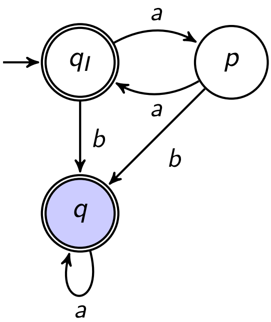
Minimum DFA M



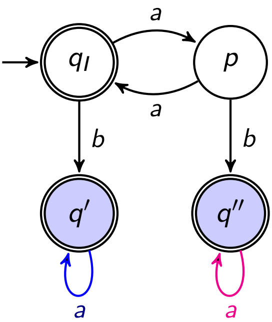
Minimal Reversible Automata

An example: $L = (aa)^* + a^*ba^*$

The idea is to use the set of states as in A' and to modify only the transitions which simulates the transitions that in M enter the state q



Minimum DFA M

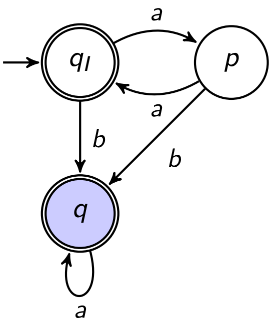


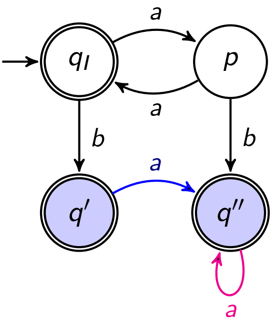
Minimal REV-DFA A'

Minimal Reversible Automata

An example: $L = (aa)^* + a^*ba^*$

The idea is to use the set of states as in A' and to modify only the transitions which simulates the transitions that in M enter the state q



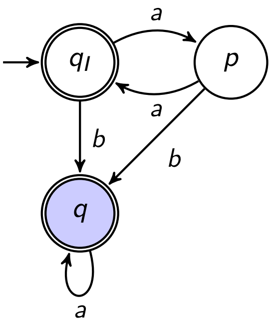


Minimum DFA M

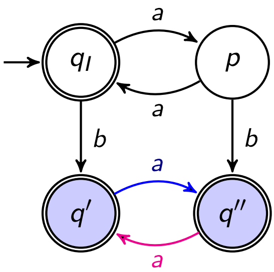
Minimal Reversible Automata

An example: $L = (aa)^* + a^*ba^*$

The idea is to use the set of states as in A' and to modify only the transitions which simulates the transitions that in M enter the state q



Minimum DFA M



Another minimal REV-DFA

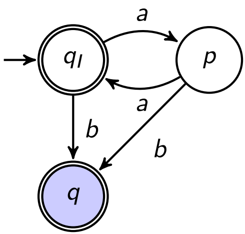
Minimal Reversible Automata

A characterization of languages having a unique minimal REV-DFA

Corollary

There exists a unique (up to isomorphism) minimal REV-DFA *accepting L if and only if for each $q \in Q$ in the irreversible part of the minimum* DFA, *all the transitions entering in q are on the same symbol.*

The minimum DFA accepting $L = (aa)^* + a^*b$:



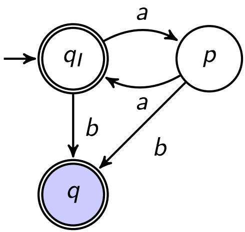
Minimal Reversible Automata

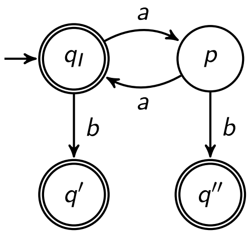
A characterization of languages having a unique minimal REV-DFA

Corollary

There exists a unique (up to isomorphism) minimal REV-DFA *accepting L if and only if for each $q \in Q$ in the irreversible part of the minimum* DFA, *all the transitions entering in q are on the same symbol.*

The minimum DFA and the *unique* minimal REV-DFA:





Minimal Reversible Automata

Observations

Minimal $\equiv$ Minimum number of states

$\Rightarrow$ Trying to merge two equivalent states in a minimal REV-DFA A, the resulting automaton is not reversible, i.e., A is *reduced*

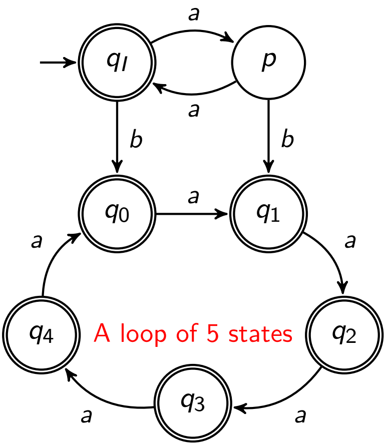
However,

there exist REV-DFAs which are reduced but not minimal!

Reduced Reversible Automata

Merging equivalent states: an example

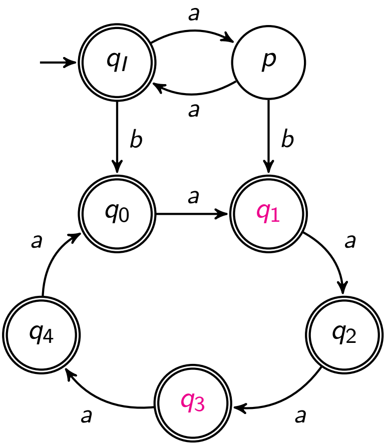
$L = (aa)^* + a^* b a^*$



All states in the loop are equivalent.
Let us try to merge two of them, i.e., q_1 and q_3

Merging equivalent states: an example

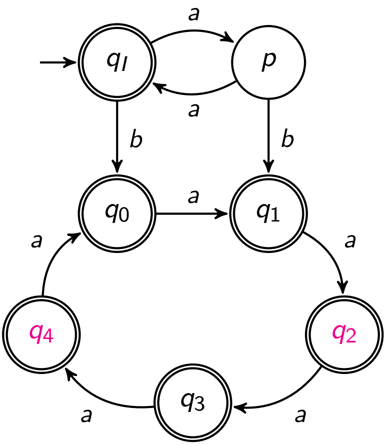
$L = (aa)^* + a^* ba^*$



q_1 - q_3

Merging equivalent states: an example

$L = (aa)^* + a^*ba^*$

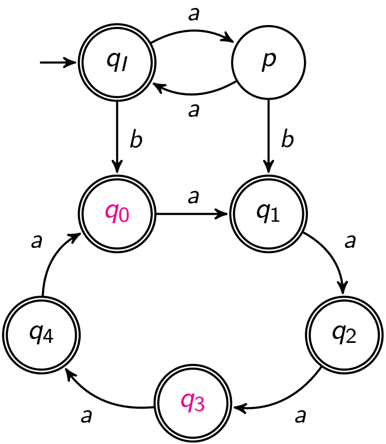


q_1 - q_3

q_2 - q_4

Merging equivalent states: an example

$L = (aa)^* + a^*ba^*$



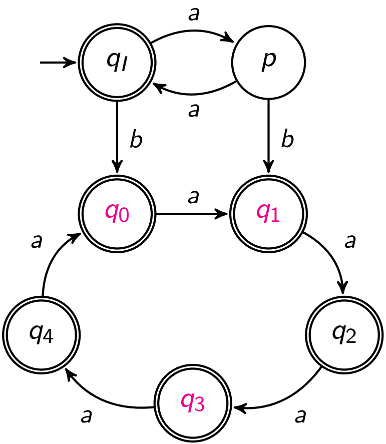
q_1 - q_3

q_2 - q_4

q_3 - q_0

Merging equivalent states: an example

$L = (aa)^* + a^*ba^*$



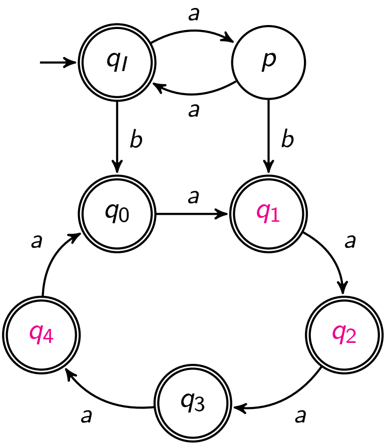
q_1 - q_3

q_2 - q_4

q_3 - q_0 - q_1

Merging equivalent states: an example

$L = (aa)^* + a^* ba^*$



q_1 - q_3

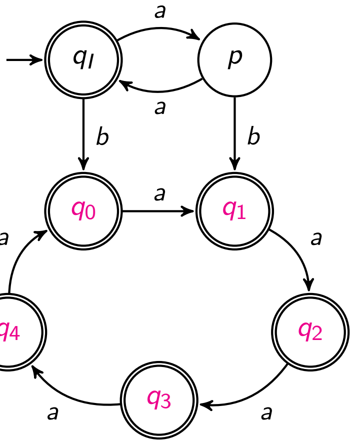
q_2 - q_4

q_3 - q_0 - q_1

q_4 - q_1 - q_2

Merging equivalent states: an example

$L = (aa)^* + a^*ba^*$



q_1 - q_3

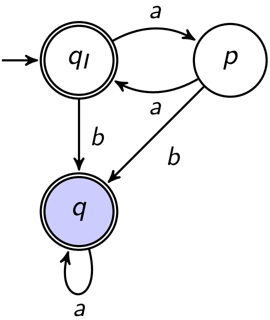
q_2 - q_4

q_3 - q_0 - q_1

q_4 - q_1 - q_2 - q_3 - q_0

Merging equivalent states: an example

$L = (aa)^* + a^*ba^*$



q_1 - q_3

q_2 - q_4

q_3 - q_0 - q_1

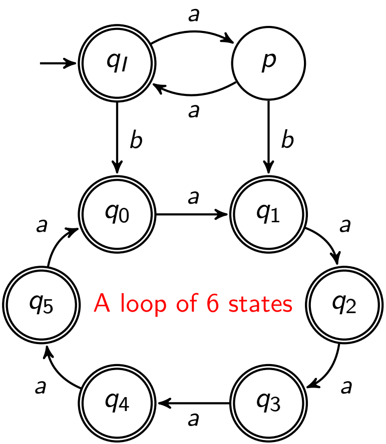
q_4 - q_1 - q_2 - q_3 - $q_0 = q$

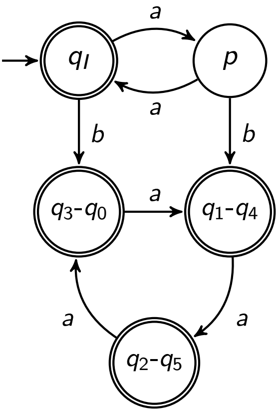
The resulting automaton
is not reversible!

A reversible automaton is said to be *reduced* if *every* automaton obtained from it by merging some equivalent states is not reversible.

Merging equivalent states: another example

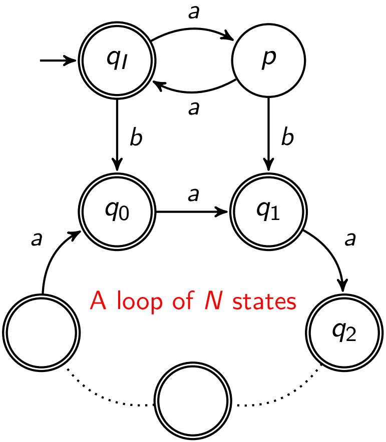
$L = (aa)^* + a^* ba^*$





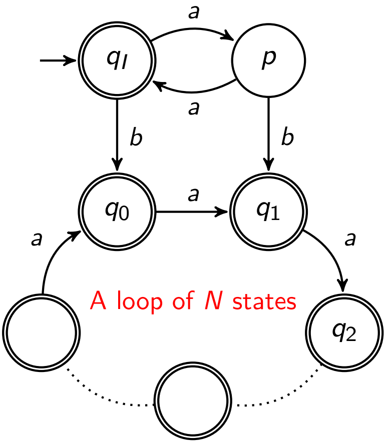
The resulting automaton
is reversible!

$L = (aa)^* + a^*ba^*$



The REV-DFA is reduced if and only if N is prime

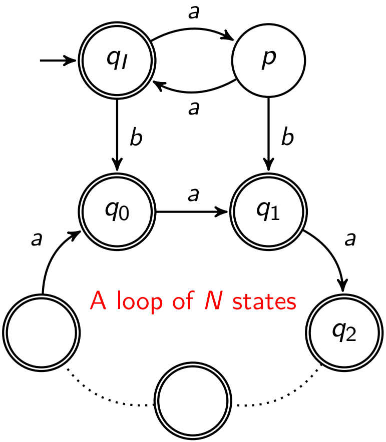
$L = (aa)^* + a^*ba^*$



- For each arbitrarily large N, there exists a reduced REV-DFA accepting L with more than N states

The REV-DFA is reduced if and only if N is prime

$L = (aa)^* + a^*ba^*$



- For each arbitrarily large N, there exists a reduced REV-DFA accepting L with more than N states
- There are infinitely many reduced equivalent REV-DFAs

The REV-DFA is reduced if and only if N is prime

What about other reversible languages?

Problem

Given a reversible language L, is it always possible to find arbitrarily large and hence infinitely many equivalent reduced reversible automata?

The answer is negative when L is finite.

What about L infinite?

What about other reversible languages?

Problem

Given a reversible language L, is it always possible to find arbitrarily large and hence infinitely many equivalent reduced reversible automata?

The answer is negative when L is finite.

What about L infinite?

Reduced Reversible Automata

A sufficient condition

- L reversible language
- $M = (Q, \Sigma, \delta, q_I, F)$ minimum DFA accepting L

Theorem

If M contains a loop in the irreversible part, then there exist infinitely many nonisomorphic reduced REV-DFA*s accepting L.*

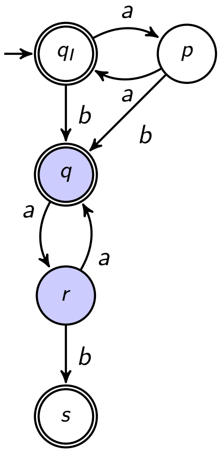
Reduced Reversible Automata

Proof idea

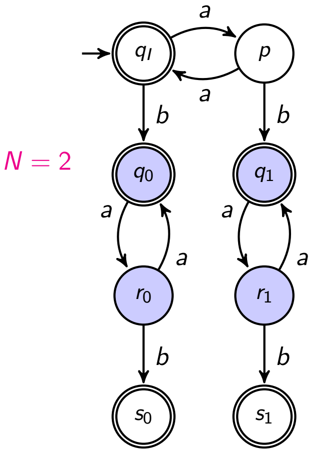
- A generalization of the argument in previous example
- Let N be a sufficient large prime number
- In a minimal REV-DFA modify the part corresponding to a nontrivial strongly connected component (SCC) $\mathcal{C}$ in the irreversible part, with N copies of each state in $\mathcal{C}$ and arranging the transitions in such a way that all the states in these N copies form a new SCC
- Furthermore, all the SCCs that follow $\mathcal{C}$ will be replicated a certain number of times

Reduced Reversible Automata: other example

$L = (aa)^* + a^*b + a^*b(aa)^*ab$



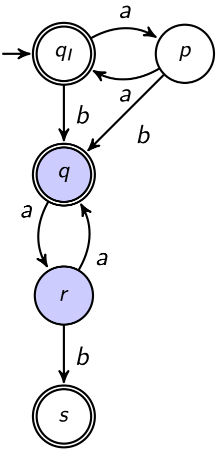
A minimum DFA



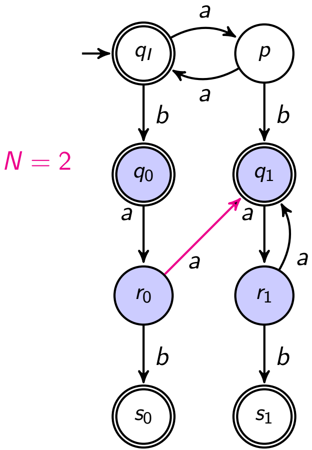
An equivalent minimal REV-DFA

Reduced Reversible Automata: other example

$L = (aa)^* + a^*b + a^*b(aa)^*ab$



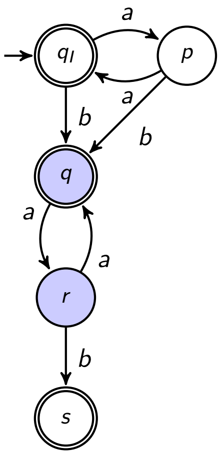
A minimum DFA



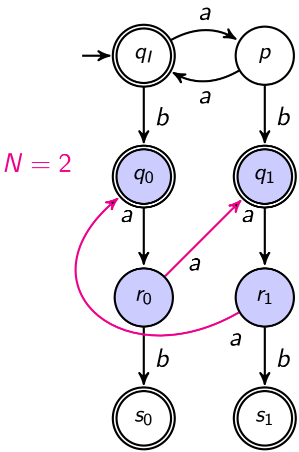
Modifying the transitions...

Reduced Reversible Automata: other example

$L = (aa)^* + a^* b + a^* b (aa)^* ab$



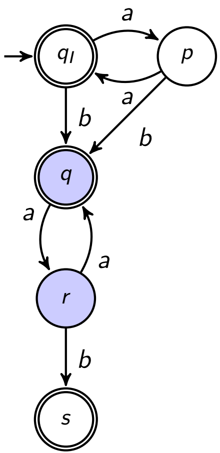
A minimum DFA



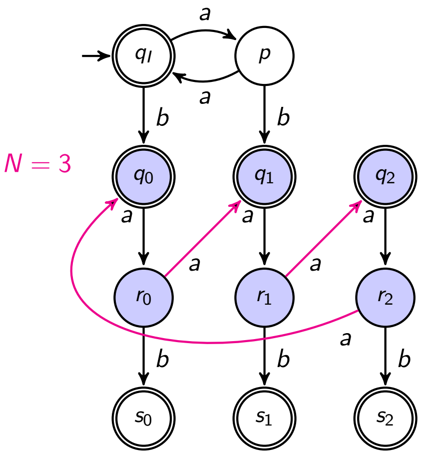
Another minimal REV-DFA for L

Reduced Reversible Automata: other example

$L = (aa)^* + a^*b + a^*b(aa)^*ab$



A minimum DFA



An equivalent reduced REV-DFA

Reduced Reversible Automata

We proved a sufficient condition:

Theorem

If M contains a loop in the irreversible part, then there exist infinitely many nonisomorphic reduced REV-DFAs *accepting L.*

The condition is not necessary

Problem: left open!

Find a characterization of reversible languages having infinitely many reduced REV-DFAs

Conclusion

Our contributions

- A characterization of the languages having several different minimal reversible automata, in terms of the structure of the minimum DFA
- A sufficient condition for the existence of infinitely many reduced REV-DFAs accepting a same reversible language

Future works

Some possible future investigations

It could be interesting to study other models:

- Unique initial state and a unique final state
- A set of initial states and a set of accepting states

Thank you for your attention!