Minimal and Reduced Reversible Automata[⋆]
(Extended Abstract)

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Abstract. A condition characterizing the class of regular languages which have several nonisomorphic minimal reversible automata is presented. The condition concerns the structure of the minimum automaton accepting the language under consideration. It is also observed that there exist reduced reversible automata which are not minimal, in the sense that all the automata obtained by merging some of their equivalent states are irreversible. Furthermore, it is proved that if the minimum deterministic automaton accepting a reversible language contains a loop in the "irreversible part" then it is always possible to construct infinitely many reduced reversible automata accepting such a language.

1 Introduction

A device is said to be *reversible* when each configuration has exactly one predecessor and one successor, thus implying that there is no loss of information during the computation. On the other hand, as observed by Landauer, logical irreversibility is associated with physical irreversibility and implies a certain amount of heat generation [7]. In order to avoid such a power dissipation and, hence, to reduce the overall power consumption of computational devices, the possibility of realizing reversible machines looks appealing.

A lot of work has been done to study reversibility in different computational devices. Just to give a few examples, in the case of general devices as Turing machines Bennet proved that each machine can be simulated by a reversible one [2], while Lange, McKenzie, and Tapp proved that each deterministic machine can be simulated by a reversible machine which uses the same amount of space [8]. As a corollary, in the case of a constant amount of space, this implies that each regular language is accepted by a *reversible two-way deterministic finite automaton*. Actually, this result was already proved by Kondacs and Watrous [4].

However, in the case of *one-way automata*, the situation is different.[1] In fact, as shown by Pin, the regular language a^*b^* cannot be accepted by any reversible

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[1] From now on, we will consider only one-way automata. Hence we will omit to specify "one-way" all the times.

automaton [10]. So the class of languages accepted by reversible automata is a proper subclass of the class of regular languages. Actually, there are some different notions of reversible automata in literature. In 1982, Angluin introduced reversible automata in algorithmic learning theory, considering devices having only one initial and only one final state [1]. On the other hand, the devices considered in [10], besides a set of final states, can have multiple initial states, hence they can take a nondeterministic decision at the beginning of the computation. An extension which allows one to consider nondeterministic transitions, without changing the class of accepted languages, has been considered by Lombardy [9], introducing and investigating *quasi reversible automata*. Classical automata, namely automata with a single initial state and a set of final states, have been considered in the works by Holzer, Jakobi, and Kutrib [5,3,6]. In particular, in [3] the authors obtained a characterization of regular languages which are accepted by reversible automata. This characterization is given in terms of the structure of the minimum deterministic automaton. Furthermore, they provide an algorithm that, in the case the language is acceptable by a reversible automaton, allows one to transform the minimum automaton into an equivalent reversible automaton, which in the worst case is exponentially larger than the given minimum automaton. In spite of that, the resulting automaton is minimal, namely there are no reversible automata accepting the same language with a smaller number of states. However, the minimal automaton is not necessarily unique, in fact there could exist different reversible automata with the same number of states accepting the same language.

We continue the investigation of minimality in reversible automata and we will refer to the following notions. Let $\mathcal{C}$ be the family of reversible automata accepting a given language L and $A \in \mathcal{C}$:

- The automaton A is *reduced* in $\mathcal{C}$ if every automaton obtained from A by merging some equivalent states does not belong to $\mathcal{C}$.
- The automaton A is *minimal* in $\mathcal{C}$ if each automaton in $\mathcal{C}$ has at least as many states as A.
- The automaton A is the *minimum* in $\mathcal{C}$ if it is the unique (up to isomorphism) minimal automaton in $\mathcal{C}$.

Our first result is a condition that characterizes languages having several different minimal reversible automata. This condition is on the structure of the transition graph of the minimum automaton accepting the language under consideration. As a special case, we show that whenever the "irreversible part" of the minimum automaton contains a loop, it is possible to construct at least two different minimal reversible automata.

We also observe that there exist reversible automata that are reduced, but not minimal. Investigating this phenomenon in detail, we were able to find a language for which there exist arbitrarily large, and hence infinitely many, reduced reversible automata. Furthermore, we obtained a general construction that allows to obtain arbitrarily large reversible automata for each language accepted by a minimum deterministic automaton satisfying the structural condition given in [3] and such that the "irreversible part" contains a loop. We know that this is also possible in other situations, namely that our condition is not necessary.

Now we introduce a few preliminary notations and notions. A *deterministic automaton* (DFA) is a tuple $A = (Q, \Sigma, \delta, q_I, F)$ with the usual meaning. We allow the transition function δ to be partial and througthout the paper, we assume that all states are useful, namely they are used to accept some word. This implies that a DFA does not contain any dead state. We denote by δ^R the *reverse* transition function that associates with each state $r \in Q$ and letter $a \in \Sigma$ the set of states from which r can be reached by reading a, i.e., $\delta^R(r, a) = \{q \in Q \mid \delta(q, a) = r\}$. A state r is said to be *irreversible* when there are at least two transitions on the same letter entering r, i.e., $\#\delta^R(r, a) \geq 2$, otherwise r is *reversible.* A DFA is said to be *reversible* (REV-DFA) when each state is reversible. A language is *reversible* when there exists a REV-DFA accepting it.

A DFA A can be split in two parts: the *reversible part* and the *irreversible part.* Roughly speaking, the irreversible part consists of all states that can be reached with a path which starts in an irreversible state, and of all transitions connecting those states. The reversible part consists of the remaining states and transitions, namely the states that can be reached from the initial state by visiting *only* reversible states, and their outgoing transitions.

The above mentioned algorithm [3] for converting a minimum irreversible DFA A into an equivalent minimal REV-DFA A', if possible, keeps the same reversible part of A and creates some copies of states and transitions in the irreversible part. However, different equivalent minimal REV-DFAs might exist. (See Figure 1).

2 Minimal Reversible Automata

In this section we present a characterization of the languages having several different minimal reversible automata. From now on, let us fix a reversible language L and the minimum DFA $M = (Q, \Sigma, \delta, q_I, F)$ accepting it.

Theorem 1. *The following statements are equivalent:*

1. *There exists a state $q \in Q$ in the irreversible part such that $\delta^R(q, a) \neq \emptyset$, $\delta^R(q, b) \neq \emptyset$, for two symbols $a, b \in \Sigma$, with $a \neq b$.*
2. *There exist at least two minimal nonisomorphic* REV-DFA*s accepting L.*

As a consequence of Theorem 1 we have the following characterization of reversible languages having a unique minimal (hence a minimum) REV-DFA:

Corollary 2. *There exists a unique (up to isomorphism) minimal* REV-DFA *accepting L if and only if for each state $p \in Q$ in the irreversible part, all the transitions entering in p are on the same symbol.*

We proved that when the minimum DFA accepting a reversible language contains a loop in the irreversible part the condition in Corollary 2 is always false, hence there exist at least two minimal nonisomorphic REV-DFAs. As a consequence, considering Corollary 2, we can observe that when a reversible language

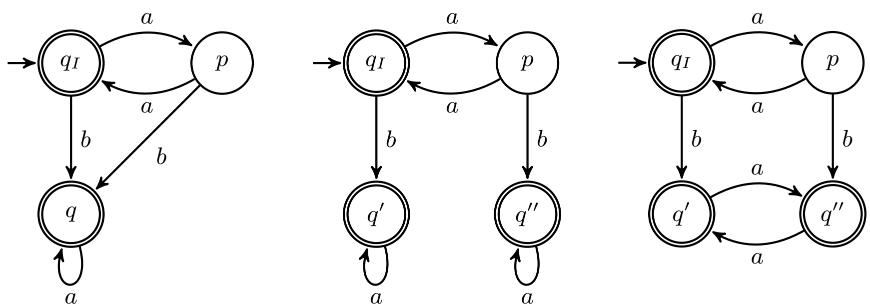

Fig. 1. A minimum DFA accepting the language $L = (aa)^* + a^*ba^*$, with two minimal nonisomorphic REV-DFAs. In the DFA on the left the reversible part consists of the states q_I and p, while the irreversible one of the state q. The REV-DFA in the center is obtained by the algorithm in [3].

has a unique minimal REV-DFA, all the loops in the minimum DFA accepting it should be in the reversible part. However, the converse does not hold, namely there are languages whose minimum DFA does not contain any loop in the irreversible part, which does not have a unique minimal REV-DFA. Indeed, in [3] an example with a finite language is presented.

3 Reduced Reversible Automata

In this section, we consider reduced REV-DFAs. There exist REV-DFAs which are reduced but not minimal. Furthermore, there exist reversible languages having arbitrarily large reduced REV-DFAs and, hence, infinitely many reduced REV-DFAs.

In Figure 2 a reduced REV-DFA equivalent to the DFAs in Figure 1 is depicted. If we try to merge two states in the loop, then the loop collapses to a single state, so producing the minimum DFA, which is irreversible. Actually, this example can be modified by using a loop of N states: if (and only if) N is prime, we get a reduced automaton. This is a special case of the construction we obtained to prove the following:

Theorem 3. *If M contains a state q in the irreversible part such that the language accepted by computations starting from q is infinite, then there exist infinitely many nonisomorphic reduced* REV-DFA*s accepting L.*

The condition in Theorem 3 is not necessary. In fact, we found an example where the minimum DFA does not contain any loop in the irreversible part, but it is possible to construct infinitely many equivalent reduced REV-DFAs.

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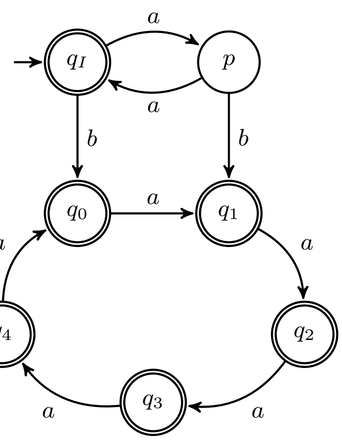

Fig. 2. A reduced REV-DFA equivalent to DFAs in Figure 1.

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